Recursive renormalization of the singlet one-pion-exchange plus point-like interactions

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Abstract

The subtracted kernel approach is shown to be a powerful method to be implemented recursively in scattering equations with regular plus point-like interactions. The advantages of the method allows one to recursively renormalize the potentials, with higher derivatives of the Dirac-delta, improving previous results. The applicability of the method is verified in the calculation of the $^1S_0$ nucleon-nucleon phase-shifts, when considering a potential with one-pion-exchange plus a contact interaction and its derivatives. The $^1S_0$ renormalization parameters are fitted to the data. The method can in principle be extended to any derivative order of the contact interaction, to higher partial waves and to coupled channels.

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1 Introduction

The pioneering work of Weinberg [1] launched the basis for the effective field theory (EFT) of nuclear forces starting from the expansion of an effective chiral Lagrangian. It gives a nucleon-nucleon (NN) interaction which is, in leading order, the one-pion-exchange potential (OPEP) plus a Dirac-delta. The program of applying effective field methods to the NN system was

pursued by many authors with significative results in few-nucleon systems (see e.g. [2]). In a more general context of few-body systems, short range interactions have many applications which are discussed in detail in ref. [3].

More recently, the authors of [4] and [5] treat the one-pion-exchange potential (OPEP) plus derivative Dirac-delta interactions using dimensional and boundary condition regularizations, respectively. In particular, when comparing the theoretical approach with the data [6], it was shown in ref. [7] that the leading order interaction, OPEP plus a Dirac-delta, renormalized using subtracted scattering equations, dominates the results obtained for the ${}^3S_1 - {}^3D_1$ phase-shifts and mixing parameter. However, considering only the leading-order term, the results obtained for the ${}^1S_0$ phase shift are not satisfactory. This is an evidence that higher order terms in the effective interaction are important in this channel (see e.g. refs. [4,5]).

The fit of the ${}^1S_0$ phase-shift up to laboratory momentum $p_{Lab} \sim 300$ MeV/c requires an effective NN interaction with the addition of a term with second order derivatives of the Dirac-delta. In the relative momentum space, we have

$$\langle \vec{p'}|V|\vec{p}\rangle = \langle \vec{p'}|V_\pi^s|\vec{p}\rangle + \sum_{i,j=0}^{1} \lambda_{ij} p'^{2i} p^{2j}, \tag{1}$$

where the λ's are unregulated strengths and $\langle \vec{p'}|V_\pi^s|\vec{p}\rangle$ is a matrix element of the one-pion-exchange potential. The motivation of the second term of (1) is to simulate effects of heavy particle exchanges by a sum of a Dirac-delta interaction and its derivatives, while keeping the OPE as the long range part of the interaction. In the renormalization method described in [7], we include λ_{11}, as the method is based on a kernel-subtraction procedure that generates terms of the type $p'^2 p^2$ in the scattering matrix elements even when one considers only p^2 and p'^2 in the interaction. In fact, the method also generates higher-order derivatives terms when OPE is considered. In the Weinberg counting rule scheme, the derivative contact interactions we are considering comes along with the two-pion-exchange (TPE) [1,13]. For a recent calculation in the nn system considering TPE see e.g. ref. [14]. As an exploratory calculation, in the present work we consider only the contact terms, leaving TPE for a future work.

In the scattering equation, the effective bare potential (1) produces integrals that diverge as much as p^5. Therefore it is necessary at least three subtractions in the kernel of the Lippman-Schwinger (LS) equation, since each subtraction introduces a factor of p^{-2}. Differently from the recent works [4] and [5], we implement the method of subtracted scattering equations [8] to deal with this problem. The subtraction method has also been shown to be practical in providing renormalization group invariant solutions for three-body scattering equations with contact interactions [9,10,11], and also proved to be useful in describing the halo structure of weakly bound exotic light nuclei [12].

The one-subtracted scattering equation used in our previous work [7] was generalized in [8] to allow any order of subtraction, permitting the inclusion of derivatives of the contact interaction in the effective two-body potential. The driving term of the n-subtracted LS equation

is constructed recursively, renormalizing the model at each subtraction order, while keeping renormalization group invariance of the approach (for details see [8]).

In the present work, we obtain the ${}^1S_0$ nucleon-nucleon amplitude from the effective interaction (1), using a three-times subtracted scattering equation, as demanded by the higher divergent term of the type p^5. We perform an analysis of the physical contribution coming from each order term in the recursive order-by-order renormalization procedure. It this way we access their significance in the parameterization of the effective interaction to obtain the desired observables.

We should note that the contact interactions that we are considering are obviously meaningless without regularization and renormalization due to the generated ultraviolet divergences in the Lippman-Schwinger equation.

This work is organized as follows. In section 2, we discuss the subtraction method applied to OPEP. In section 3, we present the main formulas of the recursive subtraction method to treat the scattering equation, as well as our strategy to solve it. In section 4, we show numerical results for the singlet phase-shift with our conclusion.

2 One-fold subtracted T-matrix equation: the one-pion-exchange potential

The partial wave decomposition of OPEP for the ${}^1S_0$ state is [15]:

$$V_{\pi,s}(p',p) = \frac{g_a^2}{16\pi f_\pi^2} - \frac{g_a^2}{32\pi f_\pi^2}\int_{-1}^{1} dx \frac{m_\pi^2}{p^2+p'^2-2pp'x+m_\pi^2} \ , \tag{2}$$

where p and p' are the relative momentum. The regular part of OPE has a finite scattering matrix, solution of the partial-wave projected LS equation:

$$T_\pi(p',p;k^2) = V_{\pi,s}^{reg}(p',p) + \frac{2}{\pi}\int_0^\infty dq q^2 \frac{V_{\pi,s}^{reg}(p',q)}{k^2-q^2+i\epsilon} T_\pi(q,p;k^2) \ , \tag{3}$$

where $V_{\pi,s}^{reg}(p',p)$ is the second term in the right-hand side of (2).

The T-matrix of the one-pion-exchange plus the Dirac-delta potential [7] is obtained here using Distorted Wave Theory [16] as suggested in [4]:

$$T_{\pi+\delta}(E) = T_\pi(E) + \left[1 + T_\pi(E) G_0^{(+)}(E)\right] T_\delta(E) \left[1 + G_0^{(+)}(E) T_\pi(E)\right] \ , \tag{4}$$

where $G_0^{(+)}(E) = [E - H_0 + i\epsilon]^{-1}$ is the free Green's function with $H_0 = p^2$ in the two-nucleon rest-frame, and

$$T_\delta(E) = V_\delta + V_\delta G_\pi^{(+)}(E) T_\delta(E) \ . \tag{5}$$

The Green's function for the regular part of the pion-exchange interaction is

$$G_\pi^{(+)}(E) = G_0^{(+)}(E) + G_0^{(+)}(E) T_\pi(E) G_0^{(+)}(E) \ . \tag{6}$$

The direct solution of (5) is plagued by infinities in the momentum integration. In order to get finite solution within our scheme we reformulate (5) in a subtracted fashion, where only one subtraction is enough [7].

Let us briefly explain the subtraction method [7,8] discussing the treatment of potentials where the matrix elements behave as a constant in the ultraviolet momentum region, like in (5), which needs one subtraction to allow finite results for the T-matrix.

Using the Lippmann-Schwinger equation

$$T(E) = V[1 + G_0^{(+)}(E) T(E)] = [1 + T(E) G_0^{(+)}(E)] V \ , \tag{7}$$

the potential V can be formally written in terms of the T-matrix at a certain energy $-\mu^2$ as

$$V = \left[1 + T(-\mu^2) G_0(-\mu^2)\right]^{-1} T(-\mu^2) \ . \tag{8}$$

For convenience we use a negative energy for the subtraction such that the Green's function is real.

¿From (8) and (7) we obtain the one-subtracted scattering equation:

$$T(E) = T(-\mu^2) + T(-\mu^2) \left(G_0^{(+)}(E) - G_0(-\mu^2)\right) T(E) \ . \tag{9}$$

By introducing a convenient notation, which is useful for more subtractions, the driving term of (9) is denoted as $V^{(1)}(-\mu^2) \equiv T(-\mu^2)$. A subtracted free Green function is written as

$$G_1^{(+)}(E; -\mu^2) \equiv G_0^{(+)}(E) - G_0(-\mu^2) = -(\mu^2 + E) G_0(-\mu^2) G_0^{(+)}(E), \tag{10}$$

which allows to rewrite the renormalized T-matrix equation (9) as

$$T(E) = V^{(1)}(-\mu^2) + V^{(1)}(-\mu^2) G_1^{(+)}(E; -\mu^2) T(E). \tag{11}$$

For a Dirac-delta potential, (11) produces finite results once $V^{(1)}(-\mu^2)$ is given. In this simple case, the matrix elements of $V^{(1)}(-\mu^2)$ in the relative momentum representation is just the renormalized coupling constant.

The method of subtracted equations is easily applied to (5), and one should note that the free Green's function in (11) is being replaced by the interacting one, G_π. We obtain the T-matrix for an arbitrary energy E in terms of the T-matrix at a given energy scale $-\mu^2$:

$$T_\delta(E) = T_\delta(-\mu^2) + T_\delta(-\mu^2)\left(G_\pi^{(+)}(E) - G_\pi(-\mu^2)\right) T_\delta(E)\ . \tag{12}$$

The renormalized strength of the interaction defines $T_\delta(E)$ at the subtraction point, $T_\delta(-\mu^2) = \lambda_{\mathcal{R}00}$, which is enough to get a finite amplitude for the OPEP plus a Dirac-delta at this energy:

$$\begin{aligned} T_{\pi+\delta}(p', p\ ;\ -\mu^2) &= T_\pi(p', p\,; -\mu^2) \\ &+ \left[1 + \frac{2}{\pi}\int_0^\infty dq\ q^2 \frac{T_\pi(p', q; -\mu^2)}{-\mu^2 - q^2}\right] \lambda_{\mathcal{R}00} \left[1 + \frac{2}{\pi}\int_0^\infty dq'\ q'^2 \frac{T_\pi(q', p; -\mu^2)}{-\mu^2 - q'^2}\right]\ , \end{aligned} \tag{13}$$

where the T-matrix for the regular part of the one-pion-exchange potential in the ${}^1S_0$ channel is given by eq. (3).

3 Subtracted T-matrix equations

The scattering T-matrix is not finite for the bare effective potential (1), which imply that regularization and renormalization are required to define the scattering amplitude. The method we use consists in constructing regularized and renormalized scattering equations with propagators subtracted at certain scales, which are convenient for introducing the physical inputs.

Our motivation in using the subtracted method was to treat a bare potential that includes a regular part plus the Dirac-delta interaction and its derivatives, for which the T-matrix has a definite form for a certain $E = -\mu^2$. That given, no approximations are performed in getting the scattering amplitude.

Let us consider point-like interactions that include up to four terms, a contact interaction plus its derivatives. After partial-wave decomposition to the singlet $s-$wave state, the bare potential is given by

$$\begin{aligned} V_s(p', p) &= V_{\pi,s}^{reg}(p', p) + \sum_{i,j=0}^{1} \lambda_{ij} p'^{2i} p^{2j} \quad (\lambda_{ij} = \lambda_{ji}^*) \\ &= \underbrace{V_{\pi,s}^{reg}(p', p) + \lambda_{00}}_{V_{\pi+\delta}} + \underbrace{\lambda_{01}p'^2 + \lambda_{10}p^2 + \lambda_{11}p'^2 p^2}_{V_{\delta'}}\ , \end{aligned} \tag{14}$$

where $V_{\pi+\delta}$ corresponds to the regular part of OPEP plus a Dirac-delta interaction V_δ.

In order to calculate the scattering amplitude for the bare potential (14) with subtracted equations, the strategy is the following. First one has to define the starting point of the iterative procedure to get the driving term of the subtracted equation. In the present case, at least three subtractions are required to allow a finite result for the the T-matrix, and two iterations are consequently needed to get the driving term. The iterative process begins with the T-matrix of OPE plus Dirac-delta potentials (13), leading to one subtraction in the kernel ($n = 1$), as shown in section 2. Next, as we show in the following subsection, two iterations are performed and added the amplitude $\lambda_{\mathcal{R}10}(p'^2 + p^2) + \lambda_{\mathcal{R}11}p'^2p^2$ at the subtraction scale ($\lambda_{\mathcal{R}10}$ and $\lambda_{\mathcal{R}11}$ are the renormalized strengths of the interaction).

3.1 Three-fold subtracted T-matrix equation

A three-fold subtracted T-matrix equation has to be used to define the scattering amplitude of the effective potential (14), which in operator form and for a general number of subtractions n reads [8]:

$$T(E) = V^{(n)}(-\mu^2; E) + V^{(n)}(-\mu^2; E)G_n^{(+)}(E; -\mu^2)T(E), \tag{15}$$

where

$$V^{(n)}(-\mu^2; E) \equiv \left[1 - (-\mu^2 - E)^{n-1}V^{(n-1)}(-\mu^2; E)G_0^n(-\mu^2)\right]^{-1} V^{(n-1)}(-\mu^2; E)\ , \tag{16}$$

$$G_n^{(+)}(E; -\mu^2) \equiv \left[(-\mu^2 - E)G_0(-\mu^2)\right]^n G_0^{(+)}(E). \tag{17}$$

One should note that the form of the above equation with $n = 1$ when applied to three-body systems results in the subtracted form used in [11], where the boundary condition was taken at the scattering threshold.

The recursive formula to derive the driving term $V^{(3)}(-\mu^2)$ starts from $V^{(1)}(-\mu^2) = T_{\pi+\delta}(-\mu^2)$ given by (13). So far, only the OPEP and the Dirac-delta interactions have been introduced in the calculation. The higher order derivatives of the Dirac-delta potential in (14) are introduced in the driving term of the three-fold subtracted equation.

The matrix element of $V^{(3)}_{\pi+\delta+\delta'}(-\mu^2)$ for the full effective interaction of (14) in the angular momentum basis,

$$V^{(3)}_{\pi+\delta+\delta'}(p', p\,; -\mu^2; k^2) = V^{(3)}_{\pi+\delta}(p', p\,; -\mu^2; k^2) + \lambda_{\mathcal{R}10}(p'^2 + p^2) + \lambda_{\mathcal{R}11}p'^2p^2\ , \tag{18}$$

get the contribution from the derivatives of the Dirac-delta interaction, through the values of $\lambda_{\mathcal{R}ij}$ - the renormalized strengths of the corresponding terms in the potential.

The integral equations for the matrix elements, $V^{(n)}_{\pi+\delta}(p', p\,; -\mu^2; k^2)$, are

$$V^{(n)}_{\pi+\delta}(p',p\ ;\ -\mu^2;k^2) = V^{(n-1)}_{\pi+\delta}(p',p\,;-\mu^2;k^2) + \frac{2}{\pi}\int_0^\infty dqq^2\left(\frac{\mu^2+k^2}{\mu^2+q^2}\right)^{n-1}\frac{V^{(n-1)}_{\pi+\delta}(p',q;-\mu^2;k^2)}{-\mu^2-q^2}V^{(n)}_{\pi+\delta}(q,p;-\mu^2;k^2), \tag{19}$$

with $k=\sqrt{E}$.

The explicit form of the three-fold subtracted LS equation is:

$$T(p',p;k^2) = V^{(3)}_{\pi+\delta+\delta'}(p',p;-\mu^2;k^2) + \frac{2}{\pi}\int_0^\infty dqq^2\left(\frac{\mu^2+k^2}{\mu^2+q^2}\right)^3\frac{V^{(3)}_{\pi+\delta+\delta'}(p',q;-\mu^2;k^2)}{k^2-q^2+i\epsilon}T(q,p;k^2)\,. \tag{20}$$

In the angular momentum basis the scattering amplitude is

$$T(k,k;k^2) = -\frac{1}{k\cot\delta - i\ k}\ , \tag{21}$$

and the low-energy parameters are defined by the effective range expansion $k\cot\delta = -1/a + (r_0/2)k^2 + \ \ldots$, with a the scattering length and r_0 the effective range.

We observe that in our method the kernel is subtracted at a given scale which allows to perform the momentum integrals up to infinity, in that sense the subtracted equations are automatically regularized. In the cutoff regularization, the Hilbert space is truncated and intermediate states goes up to a given momentum scale, such that the effective interaction includes the effect of the neglected states [17]. Our method softens the contribution of the higher momentum intermediate states, with the partially removed physics parameterized through the driving term. Of course we could move the subtraction scale, at the expense of a more complicated form for the driving term. This fact also happens in a cut-off regularization when the cut-off is moved [17].

3.2 Invariance under dislocation of the subtraction point

The nucleon-nucleon observables are invariant under the change of the arbitrary subtraction point, therefore one can start at any convenient energy scale μ^2. However, the form of the driving term and its coefficients in (18) which define the scattering amplitude are tied to the prescription used to define the renormalized theory. The key point of the renormalization group method is to change this prescription without altering the predictions of the theory [18,19].

The invariance of the T-matrix under changes of renormalization prescriptions imposes a definite rule to modify $V^{(n)}$ in (15), which appears in a form of a non-relativistic Callan-Symanzik (NRCS) equation [8,20]:

$$\frac{\partial V^{(n)}(-\mu^2;E)}{\partial \mu^2} = -V^{(n)}(-\mu^2;E)\frac{\partial G_n^{(+)}(E;-\mu^2)}{\partial \mu^2}V^{(n)}(-\mu^2;E)\ , \tag{22}$$

derived from (15) and $\frac{\partial T(E)}{\partial \mu^2} = 0$. Equation (22) substantiate the invariance of the renormalized T-matrix under dislocation of the subtraction point, with the boundary condition given by (18). Then, we observe that there is a non-trivial dependence on the subtraction point appearing in the driving term of the subtracted scattering equation, although the physical results of the model are kept unchanged.

The solution of (22) implies in a complicated evolution of $V^{(n)}$ as μ changes. Not only the lambda's would change, but also the form of the driving term. The ultraviolet behavior of the driving term is not changed by the evolution in μ. The evolution should not be truncated as μ is varied to keep the T-matrix invariant. At different μ the potential $V^{(n)}(-\mu^2;E)$ has a complicate form from the solution of NRCS equation. Similarly, the evolution of renormalization group equations as introduced by Bogner, Kuo and Schwenk [17] for the NN scattering does not truncate on certain operators as the cutoff is varied to keep observables unchanged.

4 Numerical Results and Conclusion

The renormalized strengths $\lambda_{\mathcal{R}00}$, $\lambda_{\mathcal{R}10}$, $\lambda_{\mathcal{R}11}$ and the subtracion point μ are found by fitting some low-energy phase shift data. For each set of $\lambda_{\mathcal{R}10}$, $\lambda_{\mathcal{R}11}$ and μ, we fit the singlet scattering length $a_s = -23.739$ fm through the value of $\lambda_{\mathcal{R}00}$. The three parameters left are adjusted to reproduce the Nijgemen data [6] up to the center of mass momentum p of about 300 MeV/c. We get an effective range of $r_{0,s} = 2.73$ fm compared with the value of 2.68 fm from Ref. [6]. The parameters found are displayed in Table 1, where $\mu = 214$ MeV/c corresponds to a negative subtraction energy point $\approx$ - 50 MeV. We emphasize that the results for the scattering length shown in the table correspond to the values obtained considering only the parameters found from the best fit of the singlet phase-shift using Eq.(18) as input for the three-fold subtracted scattering equation (20). Of course, from pionless EFT, we know that one could adjust the scattering length and the singlet phase-shifts at very low-energies with only contact interactions.

First, we show in Figure 1 the results of our method for only $V_{\pi+\delta}$ for the singlet and triplet channels using the one-fold subtracted T-matrix equations as performed in detail in Ref. [7], where we have used

$$V^{(1)}_{\pi+\delta}(p',p\,;-\mu^2;k^2) = V_\pi(p',p) + \lambda_{\mathcal{R}00}\ . \tag{23}$$

It is worth to mention that, for $\mu \to \infty$, $V_\pi(p',p) \to T_\pi(p',p\,;-\mu^2;k^2)$. In the present framework, we are using directly as input $T_\pi(p',p\,;-\mu^2;k^2)$ (see Eqs. (13) and (18)). The large values of μ in Fig. 1 arise due to the choice given by Eq. (23). We observe that the large μ limit converges. For the singlet channel, $V_{\pi+\delta}$ is not enough to describe the data. However, the approach works reasonably well in the spin-triplet channel when we consider Eq. (23) for the

Interaction	$\mu\lambda_{\mathcal{R}00}$	$\mu^3\lambda_{\mathcal{R}01}$	$\mu^5\lambda_{\mathcal{R}11}$	a_s [fm]
π	0	0	0	-0.878
δ	-0.1465	0	0	-0.135
δ'	0	4.7124	5.0265	-5.951
$\delta + \delta'$	-0.1465	4.7124	5.0265	-6.995
$\pi + \delta$	-0.1465	0	0	-1.250
$\pi + \delta + \delta'$	-0.1465	4.7124	5.0265	-23.739

Table 1
Dimensionless renormalized strenghts and the contributions to the scattering length.

coupled channels with $\ell = 0$ and $\ell = 2$. Therefore, we will not consider the spin-triplet channel further in the present work where we focus on the huge discrepancy of the singlet channel. We leave a detailed analysis of the coupled channels for a future work, where we also intend to include the two-pion-exchange interaction.

In Figure 2 we present the results obtained when we include $V_{\delta'}$. In order to evaluate the contributions of the different terms in the effective potential for the parameters of the best fit displayed in Table I, we calculate the $^1S_0$ phase shifts for the individual contributions of V_π, V_δ and $V_{\delta'}$, as well as for the combinations $V_{\delta+\delta'}$, $V_{\pi+\delta}$ and $V_{\pi+\delta+\delta'}$. Below $p \sim 20$ MeV/c

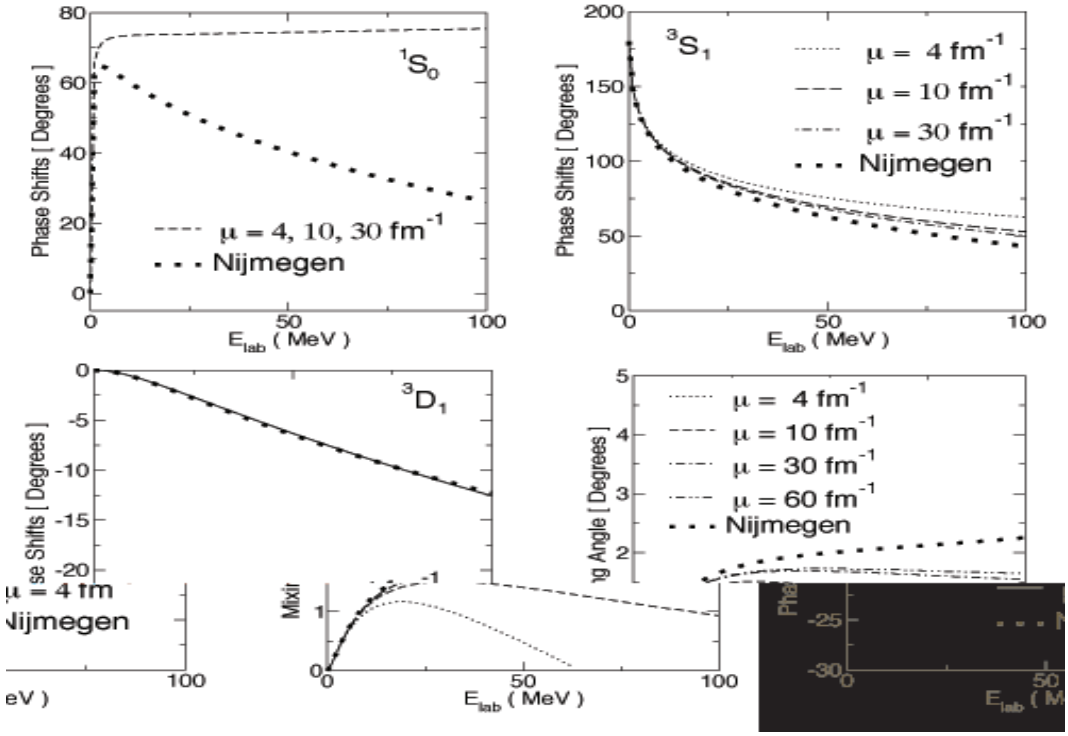

Fig. 1. Phase shifts for the singlet ($^1S_0$) and the coupled ($^3S_1 - ^3D_1$) channels for different values of μ (given in fm^{-1}) with only $V_{\pi+\delta}$. The plots were taken from our previous work [7].

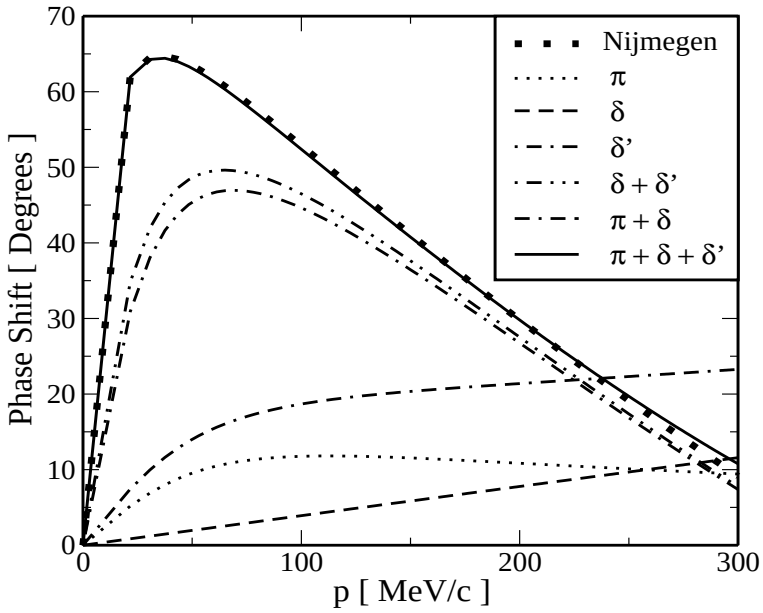

Fig. 2. Phase shifts for the singlet ($^1S_0$) for different components of the effective interaction (14) for $\mu = 214$MeV/c. The solid line is the calculation with all terms of the effective potential of (18).

the different calculations presents results that underestimate the data due to the fact that $|a_s| << 23.7$ fm, especially for V_π and $V_{\pi+\delta}$, while for $V_{\delta'}$ and $V_{\delta+\delta'}$ a_s is better approximated (see Table 1). In the last two cases, as the short-range part of the effective interaction is dominated by the higher-order derivatives of the Dirac-delta, the calculation is about enough to describe the phase shifts above $p \sim 150$ MeV/c. Therefore, the most relevant contribution in this channel comes from $V_{\delta+\delta'}$, while the regular part of the pion exchange potential appears at low energies as it provides the long range part of the potential.

Although our approach is RG invariant, it is not simple to explicitly incorporate RG invariance in the driving term of the three-fold subtracted equations. In order to have an insight of the effects included by the evolution of the driving term through the CS equation, it is important to study how our fit depends on μ. For this purpose, we calculate the phase shifts for values of μ around the best-fit value ($\mu = 214$MeV/c) keeping the λ's intact, as shown in Fig. 3. We observe that, at very low momenta the fit is not affected. For higher momenta, the phase shifts present a monotonic behaviour: it increases (decreases) as μ gets larger (smaller). The use of the NRCS equation (22) would cancel this effect in order to keep the T-matrix, and thus the observables, invariant.

In conclusion, we show in this letter how to apply the subtracted scattering equations [8] to the NN singlet channel, when more than one subtraction is required to renormalize the corresponding interaction.

The approach consists of a regularization of the integrand of the original scattering equation by using a given subtraction procedure in the propagator at some energy scale with where the T-matrix is known. The subtraction scale can be moved without modifying the calculated observables once the inhomogeneous term of the subtracted scattering equation runs with

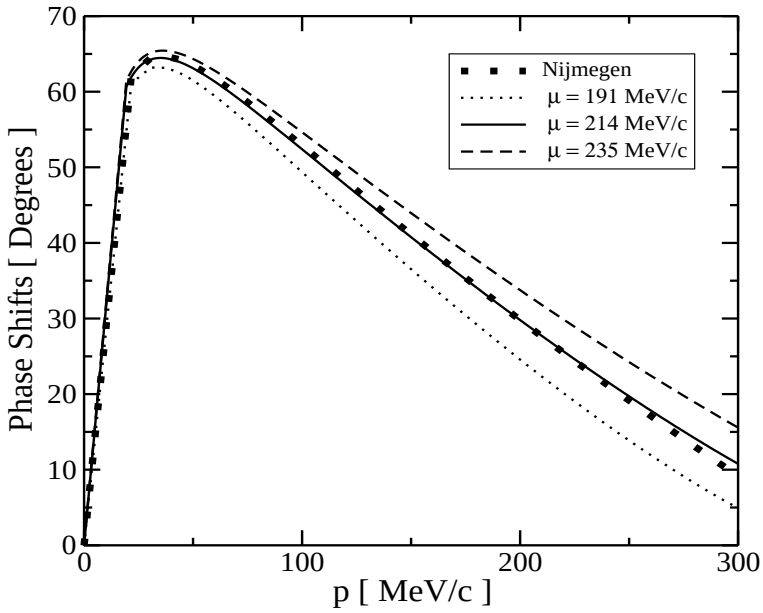

Fig. 3. The μ dependence of the singlet phase-shift for $V_{\pi+\delta+\delta'}$.

the scale according to the Non-Relativistic Callan-Symanzik (NRCS) equation. In the present case, the boundary condition is determined by the renormalized coupling constants and by the T-matrix of the OPE and Dirac-delta potentials at the specified value of the subtraction point. As a final remark, the subtracted scattering equation method is recursive and can be easily extended when higher order derivatives of the Dirac-delta are present in the two-body effective interaction.

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